Math 120A
Differential Geometry

Sample Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] Define an orientable surface.

(b) [5pts.] Using any atlas you like, show directly that the unit sphere is orientable. (You are welcome to cite computations that have been assigned previously for the atlas you choose.)

Problem 2.

(a) [5pts.] Define an allowable surface patch.

(b) [5pts.] Let $\gamma(s)$ be a regular unit-speed curve with nowhere-vanishing curvature. The tube of radius $a > 0$ around γ is parametrized by

$$\sigma(s,\theta) = \gamma(s) + a(\mathbf{n}(s)\cos\theta + \mathbf{b}(s)\sin\theta)$$

Show that σ is regular if the curvature κ of γ is less that $\frac{1}{a}$ everywhere.

Problem 3.

(a) [5pts.] Define a generalized cylinder.

(b) [5pts.] Prove that the solution set S of $3yz + 4xy + 9 = 0$ is a generalized cylinder. More specifically, what type is it?

Problem 4.

(a) [5pts.] State the isoperimetric inequality.

(b) [5pts.] Let $\gamma(t)$ be a regular simple closed curve which is area-maximizing for its length $\ell(\gamma)$. Describe the set of vertices of γ.

Problem 5.

The *genus one hyperelliptic involution* f is the map from the torus S to itself that takes $(x, y, z) \mapsto (x, -y, -z)$. Recall that one parametrization of the torus is

$$\sigma(\theta, \phi) = ((a + b\cos\theta)\cos\phi, (a + b\cos\theta)\sin\phi, b\sin\theta)$$

(a) [1pts.] How many points on the torus are fixed by f?

(b) [5pts.] Given an arbitrary point $\mathbf{p} = (x, y, z)$ on the torus, find a matrix for the linear transformation $D_{\mathbf{p}}f$ in terms of appropriate bases for $TS_{\mathbf{p}}$ and $TS_{f(\mathbf{p})}$.

(c) [4pts.] Decide whether f is a diffeomorphism, a local diffeomorphism, or neither.